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“Stokes Flow Singularity at the Junction Between Impermeable and Porous Walls”

For modeling mass transfer in lamellar membrane systems (e.g., spiral-wound modules used in desalination of seawater), a useful flow field is the classic similarity solution of Berman (1953), which assumes a uniform suction rate along the permeable boundaries and has been prolifically elaborated in the theoretical literature. More detailed treatments couple the lateral flux to the transmembrane pressure drop. In this work we consider a practically relevant entrance effect: namely, the flow singularity that arises at the abrupt transition in permeability from the sealed section of a membrane to the porous portion. On a long (outer) lengthscale the singularity appears like a discontinuity in normal velocity, characterized by a non-integrable 1/r blow-up of the pressure. A preliminary numerical solution via least-squares boundary-integral collocation reveals a continuous normal velocity coupled with a weaker, inverse-square-root singularity in pressure. Asymptotic analysis is motivated by two challenging features of the numerics: (1) the need to go to extreme distances to impose consistent outer boundary conditions, and (2) extreme mesh refinement required to resolve the inner singularity.

Typical similarity solutions (involving a stream function that varies like some power of r) are insufficient to satisfy all boundary conditions, so we generalize these by the introduction of a series in log r. By an analog of the method of reflections, outer and inner asymptotic series are developed through second order. As these do not extend far enough toward each other to enable proper asymptotic matching, we use a hybrid numerical scheme in which the outer asymptotics are iteratively patched to the inner numerics, the latter then being fitted with free coefficients of the inner asymptotic solution. A uniformly valid fit is then obtained with analytical formulas, which could serve as a basis function for numerical methods that do not require mesh refinement at the singular points.

The practical import of the pressure distribution hinges on its coupling to mass transfer in reverse osmosis. In particular, the upstream portion (nearest the singularity) is the most productive section of each membrane sheet, because solute concentration-polarization layers have not built up to the thickness that they reach farther downstream)

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