A PARAMETRIC MODEL TO ESTIMATE RISK IN A FIXED INCOME PORTFOLIO

Pilar Abad
Sonia Benito

FUNDACIÓN DE LAS CAJAS DE AHORROS
DOCUMENTO DE TRABAJO
Nº 300/2006

De conformidad con la base quinta de la convocatoria del Programa de Estímulo a la Investigación, este trabajo ha sido sometido a evaluación externa anónima de especialistas cualificados a fin de contrastar su nivel técnico.

ISBN: 84-89116-07-5

La serie **DOCUMENTOS DE TRABAJO** incluye avances y resultados de investigaciones dentro de los programas de la Fundación de las Cajas de Ahorros.
Las opiniones son responsabilidad de los autores.

A Parametric Model to Estimate Risk in a Fixed Income Portfolio[*]

Pilar Abad
D. Econometría y Estadística
Universitat de Barcelona
Diagonal, 690.
08034, Barcelona, Spain
(and Universidad de Vigo)
E-mail: pabad@uvigo.es

Sonia Benito
D. de Análisis Económico II
Universidad Nacional de Educación a Distancia
Senda del rey nº 11
28040, Madrid, Spain
E-mail: soniabm@cee.uned.es

Abstract: In this paper we propose a methodology that let us to calculate the variance and covariance matrix of a very large set of interest rate changes at a very low computational cost. The proposal uses the parametization of interest rates that underlies the model of Nelson and Siegel (1987) to estimate the yield curve. Starting with that model, we are able to obtain the variance-covariance matrix of a vector of *k* interest rates by estimating the variance of the principal components of the four parameters of the model. We used the methodology we propose to calculate risk in a fixed income portfolio, in particular to calculate Value at Risk (VaR). The results of the paper indicate that the application of our method to calculate VaR provides a precise measure of risk when compared to other parametric methods.

JEL: E43, G11.
Keywords: Value at Risk, Market risk, Nelson and Siegel method.

[*] We are grateful to Alfonso Novales for helpful comments and suggestions. Financial support from the Spanish Ministry of Education and Science and FEDER (SEJ2005-03753/ECO) and National Plan of Scientific Research, Development and Technological Innovation (BEC2003-03965) is acknowledged.

1. Introduction

One of the most important tasks facing financial institutions is the evaluation of the degree to which they are exposed to market risk. This risk appears as a consequence of the changes in the market prices of the assets that compose their portfolios. One way to measure this risk is to evaluate the possible losses that can occur from changes in market prices. This is precisely what the VaR (value at risk) methodology does. This methodology has been very widely used recently, and it has become a basic tool for market risk management of many investment banks, trading banks, financial institutions and some non-financial corporations. Also, the Basel Committee on Banking Supervision (1996) at the Bank for International Settlements uses VaR to require financial institutions such as banks and investment firms to meet capital requirements to cover the market risk that they incur as a result of their normal operations.

The VaR of a portfolio is a statistical measure that tells us what is the maximum amount that an investor may lose over a given time horizon and with a given probability. Alternatively, the VaR of a portfolio can be defined as the amount of funds that a financial institution should have in order to cover the portfolio losses in almost all circumstances, except for those that occur with a very low probability.

Although VaR is a simple concept, its calculation is not trivial. Formally, VaR (α%) is the percentil α of the probability distribution of the changes in value of a portfolio, that is, it is the value for which α% of the values lie to the left on the distribution. Consequentially, in order to calculate VaR we must firstly estimate the probability distribution of the changes in value of the portfolio.

Several methods have been developed to do this: Monte Carlo Simulation, Historical Simulation, Parametric Models, and Stress Testing. See Jorion(2000) to get a general vision of this methodologies. Among all of these, the most widely used methods are those based on the parametric approach, or on variance and co-variance. We can see some applications of this method in Morgan(1995), García-Donato at all(2001) Gento(2001), Gento(2000), Benito and Novales(2005), Alex NcMain(2001).

The parametric approach is based on the assumption that the changes in value of a portfolio will follow a known distribution, which is generally assumed to be Normal. Under such an assumption, the only relevant parameter for the calculation of VaR is the variance conditional on the changes in value of the portfolio, assuming that on average these are zero.

The estimation of this variance is not trivial, since it requires estimating the variance covariance matrix of the assets that make up the portfolio.

The estimation of this matrix poses two types of problems: (1) a dimensionality problem and (2) a viability problem. The first appears due to the large dimension of the matrix, which makes it difficult to estimate. For example, in order to estimate the variance of the return of a portfolio that is made up of five assets, it is necessary to estimate five variances and fifteen covariances, that is a total of twenty variables. This problem becomes especially important in fixed income portfolios in which the value depends on a large number of different interest rates, for different time horizons. The second problem has to do with the difficulty of estimating the conditional covariances if one uses sophisticated models, such as multivariate GARCH models. The estimation of such models is both very costly in terms of computation, and is also generally not even possible when the dimension of the matrix is greater than three. It is for this reason that these models have not been at all popular for financial management.

In the recent literature, these problems are tackled using the assumption that there exist common factors in the volatility of the interest rates, and that these same factors explain the changes in the temporal structure of the interest rates (TSIR). Under these two assumptions, it becomes theoretically possible to obtain the variance-covariance matrix of a wide range of interest rates using a factor model of TSIR. For example, Alexander (2001) and Gento (2000) show that if we begin with a principal components model (Alexander 2001) or a regression model (Gento, 2000), then we can get the variance-covariance matrix from a vector of interest rates at a low calculation cost.

The present paper proposes an alternative method of estimating the variance-covariance matrix of interest rates at a low computational cost. We take as our starting point the model of Nelson and Siegel (1987), which was developed initially to estimate the TSIR. This model provides an expression of the interest rates as a function of four parameters. Starting with this model, we can obtain the variance-covariance matrix of the interest rates by calculating the variances of only four variables – the principal components of the changes in the four parameters.

This paper continues as follows. In section 2 we present the method proposed to estimate the variance-covariance matrix for a large vector of interest rates at a low computational cost. The next sections evaluate the proposed method for a sample of data from the Spanish market. In section 3 we briefly describe the data that we use, and we apply the proposed method to obtain the variance-covariance matrix of a vector of interest rates. In section 4 we evaluate the proposed methodology to calculate the VaR in fixed income portfolios, and we compare the results with those that are obtained from standard methods of calculation. Finally, section 5 presents the main conclusions of the paper.

2. A parametric model for estimating risk.

In this section we present a methodology to calculate the variance-covariance matrix for a large vector of interest rates at a low computational cost. To do this we take as our starting point the model proposed by Nelson and Siegel (1987), designed to estimate the yield curve (TSIR).

The Nelson and Siegel formulation specifies a parsimonious representation of the forward rate function given by:

$$\varphi_m^t = \beta_0 + \beta_1 e^{\left(-\frac{m}{\tau}\right)} + \beta_2 \frac{m}{\tau} e^{\left(-\frac{m}{\tau}\right)} \tag{1}$$

This expression allows one to accommodate the different forms that may characterise (level, positive or negative slope, and greater or lower curvature) as a function of four parameters ($\beta_0,\ \beta_1, \beta_2$ and τ).

Bearing in mind the fact that the spot interest rate for a term of *m* can be expressed as the sum of the instantaneous *forward* interest rates from 0 up to *m*, that is, by integrating the expression that defines the instantaneous *forward* rate:

$$r_t(m) = \int_0^m \varphi_u^t d_u \tag{2}$$

we obtain the following expression for the spot interest rate for a term of *m*:

$$r_t(m) = \beta_0 - \beta_1 \frac{\tau}{m} e^{\left(-\frac{m}{\tau}\right)} + \beta_1 \frac{\tau}{m} + \beta_2 \frac{\tau}{m} - \beta_2 e^{\left(-\frac{m}{\tau}\right)} - \beta_2 \frac{\tau}{m} e^{\left(-\frac{m}{\tau}\right)} \tag{3}$$

Equation (3) shows that spot interest rates are a function of only four parameters. Consequentially the changes in these parameters are the variables that determine the changes in the interest rates. Using a linear approximation we can estimate the change in the zero coupon rate of term *m* from the following expression:

$$dr_t(m) \approx \begin{bmatrix} \frac{\partial r_t(m)}{\partial \beta_0} & \frac{\partial r_t(m)}{\partial \beta_1} & \frac{\partial r_t(m)}{\partial \beta_2} & \frac{\partial r_t(m)}{\partial \tau} \end{bmatrix} \begin{bmatrix} d\beta_{0,t} \\ d\beta_{1,t} \\ d\beta_{2,t} \\ d\tau_t \end{bmatrix} \tag{4}$$

In a multivariate context, the changes in the vector of interest rates that make up the TSIR can be expressed by generalizing equation (4) in the following way:

$$dr_t = G_t \, d\beta_t \tag{5}$$

where $dr_t = \left[dr_t(1) \, dr_t(2) \quad . \quad dr_t(k) \right]$, $d\beta_t^{'} = \left[d\beta_{0,t} \quad d\beta_{1,t} \quad d\beta_{2,t} \quad d\tau_t \right]$ and

$$G_t = \begin{bmatrix} \frac{\partial r_t(1)}{\partial \beta_0} & \frac{\partial r_t(1)}{\partial \beta_1} & \frac{\partial r_t(1)}{\partial \beta_2} & \frac{\partial r_t(1)}{\partial \tau} \\ \frac{\partial r_t(2)}{\partial \beta_0} & \frac{\partial r_t(2)}{\partial \beta_1} & \frac{\partial r_t(2)}{\partial \beta_2} & \frac{\partial r_t(2)}{\partial \tau} \\ . & . & . & . \\ \frac{\partial r_t(k)}{\partial \beta_0} & \frac{\partial r_t(k)}{\partial \beta_1} & \frac{\partial r_t(k)}{\partial \beta_2} & \frac{\partial r_t(k)}{\partial \tau} \end{bmatrix}.$$

This approximation (equation (5)) has been used with some success in interest rate risk management for fixed income assets (Gómez, 1999) and for portfolio immunization (Gómez, 1998).

In the context of this model, and using expression (5), we can calculate the variance-covariance matrix of a vector of changes in the k interest rates using the following expression:

$$\operatorname{var}(dr_t) = G_t \Psi_t G_t^{'} \tag{6}$$

where:

$$\Psi_t = \begin{bmatrix} \operatorname{var}(\beta_{0,t}) & \operatorname{cov}(\beta_{0,t}\beta_{1,t}) & \operatorname{cov}(\beta_{0,t}\beta_{2,t}) & \operatorname{cov}(\beta_{0,t}\tau_t) \\ & \operatorname{var}(\beta_{1,t}) & \operatorname{cov}(\beta_{1,t}\beta_{2,t}) & \operatorname{cov}(\beta_{1,t}\tau_t) \\ & & \operatorname{var}(\beta_{2,t}) & \operatorname{cov}(\beta_{2,t}\tau_t) \\ & & & \operatorname{var}(\tau_t) \end{bmatrix}$$

At this point we note that we have arrived at an important simplification in the dimension of the variance-covariance matrix that we need to estimate. Note that for a vector of k interest rates, instead of having to estimate $k(k+1)/2$ variances and covariances, we only need to estimate 10 second order moments. However, the problem associated with the difficulty of the estimation of the covariances still remains.

But we can still simplify the calculation of the variance-covariance matrix even further, by applying principal components to the vector of the changes in the parameters ($d\beta_t$). In this way, the vector of changes in the parameters of the model of Nelson and Siegel (1987) can be expressed as:

$$d\beta_t = AF_t \tag{7}$$

$$F_t = \begin{bmatrix} f_{1,t} & f_{2,t} & f_{3,t} & f_{4,t} \end{bmatrix} \text{ and } A = \begin{bmatrix} a_{\beta_0}^1 & a_{\beta_0}^2 & a_{\beta_0}^3 & a_{\beta_0}^4 \\ a_{\beta_1}^1 & a_{\beta_1}^2 & a_{\beta_1}^3 & a_{\beta_1}^4 \\ a_{\beta_2}^1 & a_{\beta_2}^2 & a_{\beta_2}^3 & a_{\beta_2}^4 \\ a_{\tau}^1 & a_{\tau}^2 & a_{\tau}^3 & a_{\tau}^4 \end{bmatrix}$$

where F_t is the vector of principal components associated with the vector $d\beta_t$ and A is the matrix of constants that form the eigenvectors associated with each one of the four eigenvalues of the variance-covariance matrix of the changes in the parameters of the Nelson and Siegel model ($d\beta_t$).

Substituting equation (7) into equation (5) and given that each principal component is orthogonal to the rest, we can express the variance-covariance matrix of the interest rates as follows:

$$\operatorname{var}(dr_t) = G_t^{*}\Omega_t G_t^{*'} \tag{8}$$

$$\text{where: } \Omega_t = \begin{bmatrix} \operatorname{var}(f_{1,t}) & 0 & 0 & 0 \\ 0 & \operatorname{var}(f_{2,t}) & 0 & 0 \\ 0 & 0 & \operatorname{var}(f_{3,t}) & 0 \\ 0 & 0 & 0 & \operatorname{var}(f_{4,t}) \end{bmatrix} \text{ and } G_t^{*} \approx G_t \times A.$$

Therefore, equation (8) gives us an alternative method to estimate the variance-covariance matrix of the changes in a vector of k interest rates using the estimation of the four principal components of the changes in the parameters of the Nelson and Siegel (1987) model. In this way, the dimensionality problem associated with the calculation of the covariances has been solved.

In the following sections we evaluate this method, both to calculate the variance matrix of a vector of interest rates, and to calculate the VaR of fixed income portfolios.

3. Estimating the variance-covariance matrix

3.1. The data

To examine the method proposed in this paper, we estimate a daily term structure of interest rates using actual mean daily Treasury transactions prices. The original data set consists of daily observations derived from actual transactions in all bonds traded on the Spanish government debt market. The database of bonds traded on the secondary market of Treasury debt covers the period from September, 1 1995 to October, 29 1997. We use this daily database to estimate the daily term structure of interest rates. We fit Nelson and Siegel's (1987) exponential model for the estimation of the yield curve and minimise price errors weighted by duration. We work with daily data for interest rates at 1, 2,…, 15 year maturities.

3.2. The results

In this section we examine this new approach to variance and covariance matrix estimation. The first section begins by comparing the changes in the estimated and observed interest rates. The changes in interest rates are modelled by equation (5), and then we compare these changes with the observed ones[1].

Then we estimate the variance-covariance matrix of a vector of 10 types of interest rate, using the methodology proposed in the previous section, and we compare these estimations (*Indirect Estimation*) with those obtained using some habitual univariate procedures (*Direct Estimation*).

Both in direct and indirect estimation we need a method for estimating variances and covariances. For the case of indirect estimation the estimation method gives us the variances of the four principal components of the changes in the parameters of the Nelson and Siegel model, which allow us to obtain, from equation (8), the variance-covariance matrix of the interest rates.

In order to estimate the variance-covariance matrix of the interest rates changes and the variance of the principal components, we use two alternative measures of volatility: exponentially weighted moving average (EWMA) and Generalized Autoregressive Conditional Heteroskedasticity models (GARCH).

(1) Under the first alternative, the variance-covariance matrix is estimated using the RiskMetrics methodology, developed by J.P. Morgan. RiskMetrics uses the so called

---

[1] The software we used in this application is MATLAB.

exponentially weighted moving average (EWMA) method. Accordingly, the estimator for the variance is:

$$\mathrm{var}(dx_t) = (1-\lambda)\sum_{j=0}^{N-1} \lambda^j (dx_{t-j} - \bar{dx})^2 \tag{9}$$

the estimator the covariance is:

$$\mathrm{cov}(dx_t dy_t) = (1-\lambda)\sum_{j=0}^{N-1} \lambda^j (dx_{t-j} - \bar{dx})(dy_{t-j} - \bar{dy}) \tag{10}$$

J.P. Morgan uses the exponentially weighted moving average method to estimate the VaR of its portfolios. On a widely diversified international portfolio, RiskMetrics found that the value $\lambda = 0.94$ with $N = 20$ produces the best backtesting results. In this paper, we use both of these values.

Therefore, we obtain the direct estimations of the variance-covariance matrix (D_EWMA) of the interest rates from equations (9) and (10) where x_t and y_t are the interest rates at different maturities . For the case of indirect estimation of the variance-covariance matrix (I_EWMA), we use equation (9) to obtain the variances of the principal components (where x_t are now these principal components) and, from there, equation (8) gives us the relevant matrix.

(2) The EWMA methodology, which is currently used for the Riskmetrics$^{\mathrm{TM}}$ data, is quite acceptable for calculating VaR measures, but some authors suggest that one alternative is to use variance-covariance matrices obtained using Multivariate Generalized Autoregressive Conditional Meteroskedasticity Models (GARCH). Nevertheless, the large variance-covariance matrices used in VaR calculations could never be estimated directly using a full multivariate GARCH model, because the computational complexity would be insurmountable. For this reason we only compute the variances of interest rates changes using univariate GARCH models and do not compute the covariance.

Given that indirect estimation (I_GARCH) does not require the estimation of covariances, we estimate the condicional variance of the principal components of the changes in the parameter of the Nelson and Siegel model using univariate GARCH models.

In the sub-section two of this section, we compare the alternative estimations of the variance-covariance matrix described above. This comparison is summarised in Table 1.

Table 1. Type of variance-covariance matrix estimation

		Type of variance models	
		EWMA	**GARCH**
Type of estimation	**Direct Estimation**	D-EWMA	~~D-GARCH~~*
	Indirect Estimation	I-EWMA	I-GARCH

* We have not estimated multivariate GARCH model because of the computational complexity are insurmountable, so that only present the result of the variances which have been estimated unsing univariate GARCH models.

What is relevant is that the estimation of the variance-covariance matrix using the methodology proposed here (indirect estimation) involves a minimum calculation cost, since it is only necessary to estimate the variance of four variables (the principal components of the daily changes in the parameters of the Nelson and Siegel model).

3.2.1. Comparing the changes of interest rates

Firstly, we have evaluated the ability of the model that we propose here to estimate the daily changes in a vector of interest rates. To do this, we compare the observed interest rates with their estimations from equation (5). In Illustration 1 in the Appendix, we show the scatter diagrams that relate the observed changes with the estimated changes in interest rates at 1, 3, 5 and 10 years. As can be seen, independently of the period considered, the relationship is very close.

In Table 2 we report some descriptive statistics of the errors of estimation of the interest rate. The average error is very small, about five basic points for all maturities. This error represents, in relative terms, 0.5% of the interest rates. Furthermore, we observe too that both the average error and the standard deviation are very similar in all period lengths so that the accurate of the model seems good for all maturities.

Table 2. Estimation errors in interest rates. Descriptive statistics.

	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
Mean [(a)]	3.1	4.3	4.8	5.0	5.1	5.0	5.0	5.0	5.0	5.0
Standard deviation	4.3	5.6	6.2	6.4	6.5	6.5	6.4	6.4	6.4	6.4
Maximum error	30.6	25.2	26.2	26.6	25.7	25.2	25.2	25.1	25.0	25.0
Minimum error	-17.5	-21.9	-24.0	-24.5	-24.4	-24.3	-24.3	-24.4	-24.5	-24.7

Note: The sample period is from 1/9/1995 to 29/10/1997. The errors (and all statistics) are expressed in basic points. (a) The average error is calculated in absolute value.

Therefore, these results imply that the degree of error committed when estimating the changes in zero coupon rates using equation (5) are practically non-existent. In what follows, we evaluate the differences in the estimation of the variance-covariance matrix using the different alternatives.

3.2.2. Comparing the estimations of variance-covariance matrix

In Illustration 2 we show the conditional variances of the interest rates at 1, 3, 5 and 10 years, as estimated using the exponentially weighted moving average method, both directly and indirectly: D_EWMA versus I_EWMA. In Illustration 3 we show the direct estimation of the conditional variances of these same interest rates using the GARCH (D_GARCH) models, and the indirect estimation of the same data (I_GARCH). As can be seen in both illustrations, in most of the time horizons considered, the variances estimated using the method proposed in this paper are very similar to the direct estimates.

The descriptive statistics of the differences between the standard deviations that are estimated using both procedures are reported in Table 3. We compare the direct and indirect estimation methods using an EWMA model in panel (a), and using a GARCH model in panel (b). Panel (a) shows that the differences in absolute value for EWMA specification oscillate between 0.62 and 1 base point. This average difference represents between 10% and 20% of the size of the estimated series.

Panel (b) of Table 3 also shows that the average difference in absolute value for GARCH specification is quite small, even though greater than those of panel (a). However, as a percentage of the estimated conditional variance series, these differences are smaller than those of panel (a). In both comparisons, we can note that the range of differences between each pair of estimations is far greater for a one year rate than for the other horizons.

We also note that the range of the estimation error is also greater for the case of one year than for the other interest rates (Table 2).

Table 3. Differences in the estimation of the standard deviation of interest rates. Descriptive statistics.

	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
	Panel (a): Comparing D_EWMA vs. I_EWMA.									
Mean (a)	0.74	0.62	0.77	0.9	0.96	0.99	1	1.01	1	0.99
Standard deviation	1.28	0.83	1.02	1.17	1.25	1.29	1.31	1.32	1.32	1.31
Maximum error	1.12	1.71	1.92	2.03	2.1	2.1	2.05	1.97	1.88	1.78
Minimum error	-14.48	-5.31	-3.12	-3.88	-4.56	-5.25	-5.61	-5.77	-5.79	-5.74
	Panel (b): Comparing D_EGARCH vs. I_EGARCH.									
Mean (a)	0.89	1.21	0.96	0.86	0.83	0.81	0.77	0.87	0.84	0.8
Standard deviation	1.35	1.1	1.29	1.28	1.24	1.28	1.25	1.26	1.25	1.23
Maximum error	17.6	5.31	7.72	7.61	7.34	8.21	7.91	7.2	7.44	7.58
Minimum error	-0.8	-3.37	-2.15	-1.61	-1.46	-1.27	-1.29	-2.58	-2.47	-2.49

Note: Sample period from 29/9/1995 to 29/10/1997 (515 observations). I_EWMA indirect estimation (equation (8)) and D_EWMA direct estimation. Riskmetrics methodology (EWMA). I_GARCH: indirect estimation (equation (8)) and D_GARCH direct estimation. Conditional autoregressive volatility models (GARCH). (a) The average of the differences has been calculated in absolute value. Differences measured in base points.

We now compare the covariances estimated directly with those obtained from the procedure suggested in this paper. As we have mentioned above, given the extreme complexity of the GARCH multivariate model estimations, the direct estimation of the covariances was only done using EWMA models.

Illustration 4 shows the estimated covariances between the different pairs of interest rates, using both procedures: D_EWMA versus I_EWMA. As can be seen in the graphs, the estimated covariances have very similar behaviour, although we can note that there are greater differences than for the variances. In Table 4 we report some of the descriptive statistics of the estimated covariances. The average absolute value difference is very small, between 0.0007 and 0.0019. However, this does represent about 40% of the average estimated covariance.

Table 4. Differences in the estimation of covariances between interest rates. Descriptive statistics.

Comparing D_EWMA vs. I_EWMA

	1 year			3 years		5 years
	3 years	**5 years**	**10 years**	**5 years**	**10 years**	**10 years**
Mean [(a)]	0.0008	0.0007	0.0007	0.0019	0.0018	0.0017
Standard deviation	0.0013	0.0013	0.0015	0.0020	0.0021	0.0022
Maximum error	0.0042	0.0078	0.0131	0.0032	0.0037	0.0046
Minimum error	-0.0140	-0.0120	-0.0138	-0.0162	-0.0170	-0.0172

Note: Sample period from 29/9/1995 to 29/10/1997 (515 observations). I_EWMA indirect estimation (equation (8)) and D_EWMA direct estimation. Riskmetrics methodology (EWMA). [(a)] The average difference is calculated in absolute value.

To sum up this section, we have shown that the procedure proposed in this paper to estimate the variance-covariance matrix of a large vector of interest rates generates results that are quite satisfactory, above all as far as variances are concerned. For the case of co-variances, we have detected some differences that could be important. In the following section we evaluate whether these differences are important for risk management. To do this, we apply the methodology to the calculation of Value at Risk (VaR) in several fixed income portfolios.

4. **Estimating the Value at Risk**

In this section we evaluate the utility of the proposed method for risk management of fixed income portfolios, by constructing a parametric measure of VaR as an indicator of the risk of a given portfolio.

4.1. Value at Risk

The VaR of a portfolio is a measure of the maximum loss that the portfolio may suffer over a given time horizon and with a given probability. Formally, the VaR measure is defined as the lower limit of the confidence interval of one tail:

$$\Pr\left[\Delta V_t(\tau) < VaR_t\right] = \alpha \tag{10}$$

where α is the level of confidence and $\Delta V_t(\tau)$ is the change in the value of the portfolio over the time horizon τ.

The methods that are based on the parametric, or variance-covariance, approach start with the assumption that the changes in the value of a portfolio follow a Normal distribution. Assuming that the average change is zero, the VaR for one day of portfolio *j* is obtained as:

$$VaR_{j,t}(\alpha\%) = \sigma_{t,dV_j} \cdot k_{\alpha\%} \tag{11}$$

where $k_{\alpha\%}$ is the α percentile of the Standard Normal distribution, and the parameter to estimate is the standard deviation conditional upon the value of portfolio *j* (σ_{t,dV_j}).

In a portfolio that is made up of fixed income assets, the duration can be used to obtain the variance of the value of portfolio *j* from the variance of the interest rates in the following way (Jorion, 2000):

$$\sigma^2_{t,dV_j} = D_{j,t}\,\Sigma_t\,D'_{j,t} \tag{12}$$

where Σ_t is the variance-covariance matrix of the interest rates and $D_{j,t}$ is the vector of the duration of portfolio *j.* This vector represents the sensitivity of the value of the portfolio to changes in the interest rates that determine its value.

In this section, value at risk measures are calculated and compared. In the parametric approach, we use the estimations of the variance-covariance matrix as obtained in the previous section (see Table 1). Table 5 illustrates the four measures of VaR that we obtain from the four variance-covariance models:

Table 5. Type of VaR measures

	Type of variance- covariance matrix estimation	Type of VaR measure
Direct Estimation	D_EWMA	VaR_D_EWMA
	D_GARCH	VaR_D_GARCH*
Indirect Estimation	I_EWMA	VaR_I_EWMA
	I_GARCH	VaR_I_GARCH

* We did not compute VaR_D_GARCH because of the impossibility to estimate a multivariate GARCH model with 10 variables.

In the case of the first VaR measure, VaR_D_EWMA, the VaR is obtained by directly estimating Σ_t with an EWMA model. This is a popular approach to measuring market risk, and it is used by JP Morgan (RiskMetric^{TM}). The second VaR measure, VaR_D_GARCH, is also obtained by directly estimating the variance-covariance matrix, but in this case the second order moments are estimated using GARCH models. This VaR measure has not been calculated, given that the large variance-covariance matrices used in VaR calculations could never be estimated directly using a full multivariate GARCH model, because the computational complexity would be insurmountable.

The final two VaR measures are calculated by estimating the variance-covariance matrix of the interest rates using the procedure described in Section 2. We can estimate the variance-covariance matrix of interest rates indirectly, by substituting equation (8) into equation (12) to obtain a new expression for the variance of the changes in the value of the portfolio:

$$\sigma^2_{t,dV_j} = D_{j,t}\, G^*\, \Omega_t\, G^{*'}\, D^{'}_{j,t} = D^m_{j,t}\, \Omega_t\, D^{m'}_t \tag{13}$$

In indirect estimation, Ω_t is a diagonal matrix that contains on its principal diagonal the conditional variance of the principal components of the changes in the four parameters of the Nelson and Siegel model, and $D^m_{j,t}$ is the modified vector of durations of portfolio *j* (of dimension 1x4) which represents the sensitivity of the value of the portfolio to changes in the principal components of the four parameters of the Nelson and Siegel model. In the VaR_I_EWMA, we use an EWMA model to estimate the variance of the principal components; and we use a GARCH model to estimate these variances in the case of the calculation of the VaR_I_GARCH measure.

4.2. The portfolios

In order to evaluate the procedure proposed in this paper for calculating VaR we have considered 4 different portfolios made up of theoretical bonds with maturities at 3, 5, 10 and 15 years, constructed from real data from the Spanish debt market. In each portfolio, the bond coupon is 3.0%. The period of analysis is from 29/9/1995 to 29/10/1997, which allows us to perform 516 estimations of daily VaR for each portfolio.

In order to estimate the daily VaR we have assumed that the characteristics of each portfolio do not change over the dates of the period of analysis: the initial value of the portfolio, the maturity date and the coupon rate. In this way, the results are comparable over the entire period of analysis since we avoid both the pull to par effect (the value of the bonds tends to par as the maturity date of the bond approaches) and the roll down effect (the volatility of the bond decreases over time).

4.3. Comparing VaR measures

In this section value at risk measures are compared. For all portfolio considered we calculate daily VaR at a 5%, 4%, 3%, 2% and 1% confidence level. Firstly, before formally evaluating the precision of the VaR measures under comparison, we examine actual daily portfolio value changes as implied by daily fluctuations in the zero cupon interest rate and compare them with the 5% VaR. In Illustration 5 we show the actual change in a 10 year portfolio together with the VaR at 5% for the three measures of VaR that we consider: VaR_D_EWMA (Figure 1), VaR_I_EWMA (Figure 2) and VaR_I_GARCH (Figure 3). In Figures 1 and 2 we observe that the value of the portfolio falls below the VaR on more occasions than in Figure 3. In all case, the number of times that the value of the portfolio falls below the VaR is closer to its theoretical level. This result is also evident in the other portfolios that we consider, but that we have not reported due to space considerations. This preliminary analysis suggests that the estimations of VaR that are obtained from both models, both directly and indirectly are very precise. However, a more rigorous evaluation of the precision of the estimations is required.

We then compare VaR measures the actual change in portfolio value on day t+1, denoted as ΔV_{t+1}. If $\Delta V_{t+1} <$ VaR, then we have an exception. For testing purposes, we define the exception indicator variable as

$$I_{t+1} = \begin{cases} 1 & if\ \Delta V_{t+1} < VaR \\ 0 & if\ \Delta V_{t+1} \geq VaR \end{cases} \tag{14}$$

a) Testing the Level

The most basic test of a value at risk procedure is to see if the stated probability level is actually achieved. The mean of the exception indicator series is the level of the procedure that is achieved. If we assume the probability of an exception is constant, then the number of exceptions follows the binomial distribution. Thus it is possible to form confidence intervals for the level of each VaR measure (see Kupiec (1995)).

Table 6. Testing the Level

VaR measures	Number of exceptions 3- years	 5-years	 10-years	 15-years	Confidence intervals at the 95% level
VaR_D_EWMA (1%)	5	5	8	10	(1 - 10)
VaR_D_EWMA (2%)	6	10	12	12	(5 - 17)
VaR_D_EWMA (3%)	10	11	14	14	(8 - 23)
VaR_D_EWMA (4%)	14	16	18	18	(12 - 30)
VaR_D_EWMA (5%)	21	23	22	24	(17 - 36)
VaR_I_EWMA (1%)	6	9	9	11	(1 - 10)
VaR_I_EWMA (2%)	7	14	14	14	(5 - 17)
VaR_I_EWMA (3%)	17	16	18	19	(8 - 23)
VaR_I_EWMA (4%)	19	24	23	24	(12 - 30)
VaR_I_EWMA (5%)	21	29	29	28	(17 - 36)
VaR_I_GARCH (1%)	3	5	5	4	(1 - 10)
VaR_I_GARCH (2%)	5	9	7	5	(5 - 17)
VaR_I_GARCH (3%)	7^*	9	13	11	(8 - 23)
VaR_I_GARCH (4%)	11^*	13	14	14	(12 - 30)
VaR_I_GARCH (5%)	13^*	17	17	17	(17 - 36)

Note: Sample period 29/9/1995 to 29/10/1997. Confidence intervals derived from the number of exceptions follows the binomial distribution (516, x%) for x=1, 2, 3, 4 and 5. An * indicates the cases in which the number of exceptions is out of the confidence interval, so that, we obtain evidence to reject the null hypothesis at the 5% level type I error rate.

Table 6 shows the level that is achieved and a 95% confidence interval for each of the 1-day VaR estimates. An * indicates the cases in which the number of exceptions is out of the confidence interval, so that, we obtain evidence to reject the null hypothesis at the 5% confidence level. For the three measures and almost all portfolios considered, the number of exceptions is inside the interval confidence, so that the VaR estimation (direct and indirect) seems to be good.

We find just only three cases in which the number of exceptions is out of the confidence interval. This happen for VaR_I_GARCH measure for 3%, 4% and 5% confidence level of the portfolio at 3 years. In those cases the number of exceptions are much lower than the theoretical level, so that it seems that this measure is overestimating the risk of short-term portfolio.

b) *Testing Consistency of Level*

We want the level of the VaR that is found to be the stated level on average, but we also want to find the stated level at all points in time. One approach to testing the consistency of the level is to use the Ljung-Box portmanteau test (Ljung and Box, 1978) on the exception indicator variable of zeros and ones. When using Ljung-Box tests, there is a choice of the number of lags in which to look for autocorrelation. If the test uses only a few lags but autocorrelation occurs over a long time frame, the test will miss some of the autocorrelation. Conversely should a large number of lags be used in the test when the autocorrelation is only in a few lags, then the test won't be as sensitive as if the number of lags in the test matched the autocorrelation.

Different lags have been used for each estimate in order to try to get a good idea of the autocorrelation. Table 7 shows the Ljung-Box statistics at lags of 4 and 8.

Table 7. Testing Consistency of Level

	Lags	3- years	5-years	10-years	15-years
VaR_D_EWMA (1%)	**4**	0,20 (0,995)	0,20 (0,995)	0,52 (0,971)	0,82 (0,936)
	8	0,41 (1,000)	0,41 (1,000)	1,06 (0,998)	5,07 (0,750)
VaR_D_EWMA (2%)	**4**	0,29 (0,990)	0,82 (0,936)	1,19 (0,879)	1,19 (0,879)
	8	0,59 (1,000)	4,98 (0,760)	13,60* (0,093)	4,10 (0,848)
VaR_D_EWMA (3%)	**4**	4,13 (0,389)	1,00 (0,910)	1,64 (0,802)	2,30 (0,681)
	8	4,85 (0,773)	4,46 (0,814)	11,22 (0,189)	5,41 (0,713)
VaR_D_EWMA (4%)	**4**	2,96 (0,565)	2,16 (0,707)	2,30 (0,681)	5,29 (0,258)
	8	5,26 (0,729)	4,41 (0,819)	7,30 (0,504)	7,17 (0,518)
VaR_D_EWMA (5%)	**4**	2,66 (0,617)	2,16 (0,707)	4,44 (0,349)	6,04 (0,196)
	8	10,66 (0,222)	6,59 (0,581)	7,90 (0,444)	7,33 (0,502)
VaR_I_EWMA (1%)	**4**	0,29 (0,990)	0,66 (0,956)	0,66 (0,956)	3,37 (0,498)
	8	0,59 (1,000)	5,91 (0,657)	5,91 (0,657)	6,77 (0,562)
VaR_I_EWMA (2%)	**4**	0,40 (0,983)	2,30 (0,681)	2,30 (0,681)	8,57* (0,073)
	8	0,80 (0,999)	5,27 (0,728)	4,63 (0,796)	10,90 (0,207)
VaR_I_EWMA (3%)	**4**	2,20 (0,700)	2,16 (0,707)	4,84 (0,305)	4,28 (0,369)
	8	7,79 (0,454)	4,34 (0,826)	6,71 (0,568)	5,47 (0,706)
VaR_I_EWMA (4%)	**4**	2,44 (0,656)	1,28 (0,864)	3,30 (0,509)	4,03 (0,403)
	8	6,75 (0,564)	2,62 (0,956)	4,48 (0,812)	6,82 (0,557)
VaR_I_EWMA (5%)	**4**	1,93 (0,748)	0,75 (0,945)	3,53 (0,474)	3,78 (0,436)
	8	5,55 (0,698)	2,49 (0,962)	7,87 (0,447)	14,77* (0,064)
VaR_I_GARCH (1%)	**4**	0,07 (0,999)	0,20 (0,995)	0,20 (0,995)	0,13 (0,998)
	8	0,15 (1,000)	0,41 (1,000)	0,41 (1,000)	0,26 (1,000)
VaR_I_GARCH (2%)	**4**	0,20 (0,995)	0,66 (0,956)	0,40 (0,983)	0,20 (0,995)
	8	0,41 (1,000)	5,91 (0,657)	9,68 (0,288)	19,69* (0,012)
VaR_I_GARCH (3%)	**4**	0,40 (0,983)	0,66 (0,956)	2,51 (0,643)	3,37 (0,497)
	8	0,80 (0,999)	5,91 (0,657)	6,15 (0,630)	9,15 (0,330)
VaR_I_GARCH (4%)	**4**	1,00 (0,910)	1,40 (0,843)	2,29 (0,682)	2,29 (0,682)
	8	4,40 (0,820)	5,04 (0,753)	5,28 (0,727)	5,93 (0,655)
VaR_I_GARCH (5%)	**4**	2,51 (0,643)	1,95 (0,746)	2,19 (0,701)	1,95 (0,745)
	8	5,05 (0,752)	3,66 (0,886)	3,91 (0,865)	3,67 (0,886)

Note: Sample period 29/9/1995 to 29/10/1997. The Ljung-Box Q-statistics on the exception indicator variable and their p-values. The Q-statistic at lag 4 (8) for the null hypothesis that there is no autocorrelation up to order 5 (10). An * indicates that there is evidence to reject the null hypothesis at the 5% level type I error rate.

We only detect the existence of autocorrelation in the portfolios at 10 years with the VaR_D_EWMA (2%) estimate, in the portfolio at 15 years with the measures VaR_I_EWMA (2%) and (5%) and the portfolio at 15 years with the measures VaR_I_GARCH (2%). In general, the results of the Ljung-Box comparison indicate that autocorrelation is not present. When we consider other lags, that are not reported here in the interests of space, the result are pretty the same, so that the VaR estimate also seems to be good using this test.

c) *Unconditional Coverage Tests*

Assuming that a set of VaR estimates and their underlying model are accurate, the exceptions can be modeled as independent draws from a binomial distribution with a probability of occurrence equal to α percent. Accurate VaR measures should exhibit the property that their unconditional coverage $\hat{\alpha} = x/T$ equals α percent, where *x* is the number of exceptions and *T* the number of observations. The likelihood ratio statistic for testing whether $\hat{\alpha} = \alpha$ is

$$LR = 2\left[\log\left(\hat{\alpha}^{x}\left(1-\hat{\alpha}\right)^{T-x}\right) - \log\left(\alpha^{x}\left(1-\alpha\right)^{T-x}\right)\right]$$

which has an asymptotic χ^2 (1) distribution.

Table 8. Unconditional Coverage Tests and The Back-testing Criterion

	% of exceptions $^{(a)\,(b)}$			
	3- years	**5-years**	**10-years**	**15-years**
VaR D EWMA (1%)	1,0%	1,0%	1,6%*	1,9%$^{*+}$
	(0,002)	(0,002)	(0,587)	(1,563)
	[-0,071]	[-0,071]	[1,257]	[2,141]
VaR D EWMA (2%)	1,2%*	3,1%	3,9%$^{+}$	4,3%$^{+}$
	(0,942)	(-1,156)	(-1,565)	(-2,340)
	[-1,358]	[1,786]	[3,044]	[3,673]
VaR D EWMA (3%)	1,9%*	2,1%*	2,7%	2,7%
	(0,990)	(0,644)	(0,065)	(0,065)
	[-1,414]	[-1,156]	[-0,382]	[-0,382]
VaR D EWMA (4%)	2,7%*	3,1%*	3,5%	3,5%
	(1,086)	(0,510)	(0,159)	(0,159)
	[-1,492]	[-1,042]	[-0,593]	[-0,593]
VaR D EWMA (5%)	4,1%*	4,5%	4,3%	4,7%
	(0,435)	(0,144)	(0,269)	(0,059)
	[-0,970]	[-0,566]	[-0,768]	[-0,364]
VaR I EWMA (1%)	1,2%	1,7%*	1,7%*	2,1%$^{*+}$
	(0,057)	(1,026)	(1,026)	(2,189)
	[0,372]	[1,699]	[1,699]	[2,584]
VaR I EWMA (2%)	1,4%*	2,7%*	2,7%*	2,7%*
	(0,533)	(0,524)	(0,524)	(0,524)
	[-1,044]	[1,157]	[1,157]	[1,157]
VaR I EWMA (3%)	3,3%	3,1%	3,5%	3,7%*
	(0,065)	(0,008)	(0,175)	(0,335)
	[0,392]	[0,134]	[0,650]	[0,908]
VaR I EWMA (4%)	3,7%	4,7%	4,5%	4,7%
	(0,061)	(0,236)	(0,118)	(0,236)
	[-0,368]	[0,755]	[0,530]	[0,755]
VaR I EWMA (5%)	4,1%*	5,6%	5,6%	5,4%
	(0,435)	(0,175)	(0,175)	(0,084)
	[-0,970]	[0,646]	[0,646]	[0,444]
VaR I GARCH (1%)	0,6%*	1,0%	1,0%	0,8%
	(0,467)	(0,002)	(0,002)	(0,124)
	[-0,956]	[-0,071]	[-0,071]	[-0,513]
VaR I GARCH (2%)	1,0%*	1,7%	1,4%*	1,0%*
	(1,498)	(0,078)	(0,533)	(1,498)
	[-1,673]	[-0,415]	[-1,044]	[-1,673]
VaR I GARCH (3%)	1,4%$^{*+}$	1,7%*	2,5%	2,1%*
	(2,602)	(1,425)	(0,188)	(0,644)
	[-2,188]	[-1,672]	[-0,640]	[-1,156]
VaR I GARCH (4%)	2,1%$^{*+}$	2,5%*	2,7%*	2,7%*
	(2,441)	(1,467)	(1,086)	(1,086)
	[-2,166]	[-1,716]	[-1,492]	[-1,492]
VaR I GARCH (5%)	2,5%$^{*+}$	3,3%*	3,3%*	3,3%*
	(3,522)	(1,552)	(1,552)	(1,552)
	[-2,585]	[-1,778]	[-1,778]	[-1,778]

Note: Sample period 29/9/1995 to 29/10/1997. (a) Between parentheses Unconditional Coverage Tests: The LR statistic for testing whether the percentage of exceptions ($\hat{\alpha} = x/T$) is α percent. An * indicates that there is evidence to reject the null hypothesis at the 5% level type I error rate. (b) Between square brackets Back-testing Criterion: The Z statistic for determining the significance of departure for $\hat{\alpha} = x/T$ from α%. An + indicates that there is evidence to reject the null hypothesis at the 5% level type I error rate.

Table 8 reports the percentage of exceptions observed for the 1%, 2%, 3%, 4% and 5% quantiles over the entire sample period. In parentheses, Table 8 reports the LR statistic for testing whether the percentage of exceptions is the quantile. For the case of measures obtained from VaR_ I_ EWMA, independently of the quantile considered, we reject the null hypothesis that the percentage of exceptions coincides with the corresponding quantile in 40% of the cases. The result with VaR_I_EWMA measure are pretty the same (45%). It is worth to note that the measure VaR_D_EWMA produce a slight underestimation the risk for 1% confidence level and overestimate the risk for 2%, 3%, 4% and 5% confidence level. However, the measure VaR_I_EWMA underestimate the risk for 1%, 2% and 3% and overestimate for 5% confidence level.

The result are worst with the measure VaR_I_GARCH. For this measure reject the null hypothesis in 75% of the times. With this measure overestimating the risk in all cases. It seems that whem the aim is to calculate value at risk, the GARCH models doesn't produce a good estimation of the volatility.

d) *The Back-testing Criterion*

The back-testing criterion is used to evaluate the performance of these VaR measures. The most popular back-testing measure for accuracy of the quantile estimator is the percentage of returns falling below the quantile estimate, denoted as $\hat{\alpha}$. For an accurate estimator of an α quantile, $\hat{\alpha}$ will be very close to α%. In order to determine the significance of departure of $\hat{\alpha}$ from α%, the following test statistic is used[2]:

$$\boldsymbol{Z} = (T\hat{\alpha} - T\alpha\%) \Big/ \sqrt{T\alpha\%(1-\alpha\%)} \xrightarrow{d} N(0,1)$$

where T is the simple size.

Table 8 presents the Z statistic for VaR measures in square brackets. For the case of measures obtained from EWMA, (independently of the quantile considered), we reject the null hypothesis that the percentage of exceptions coincides with the corresponding quantile in three cases with the VaR_D_EWMA measure and one time with the VaR_I_EWMA measure. On the other hand, with this test just only in three cases of the VaR_I_GARCH measure reject the hypothesis.

In summary, we can say that the VaR measures we obtain using the simplification proposed in this paper are so good as that we obtain from Riskmetrics method

(VaR_D_EWMA). The advantage of the method we propose is that the computational cost to calculate value at risk is much lower. Additionally, we find slight evidence that to estimate value at risk the EWMA model seem more accurate that the autoregressive conditional volatility models (GARCH models).

5. Conclusion

In this paper we propose a method for calculating the variance-covariance matrix of a large set of interest rates with a low computational cost. The methodology suggested exploits the parametrization of the underlying interest rates that was proposed by Nelson and Siegel (1987) for estimating the yield curve. The method proposed in this paper turns out to be useful for estimating VaR, since it simplifies considerably the calculation of this measure.

Following the papers of Alexander (2001) and Gento (2000), the starting point for our method is an explanatory model of the interest rates. However, contrary to those authors, our model is based on that of Nelson and Siegel (1987) whose objective was to estimate the TSIR. Using a linear approximation, this model provides a relationship in which the changes in interest rates are a function of the changes in four parameters. Although this approximation reduces the dimension of the variance-covariance matrix, it still requires covariances to be estimated. In order to solve this problem, we propose the application of principal components of the changes in the four parameters of the Nelson and Siegel model (1987). Given the orthogonality of the principal components among themselves, the resulting variance-covariance matrix has a smaller dimension since it is diagonal, that is, all the covariances are zero.

The procedure that we propose in this paper has been contrasted using data from the Spanish debt market. The results of this application of our methodology are very satisfactory. On the one hand, the variances that we estimate with our procedure and those that are given by a direct estimation are quite similar, independently of the method used to estimate them (Exponentially Weight Moving Average Model (EWMA) vs. autoregressive conditional volatility models).

Concerning the calculation of VaR, the estimations that we obtain using EWMA models, both under direct and indirect estimation (following the procedure proposed here) are quite precise. The estimations of VaR get worse when we directly estimate the variance-

---

[2] This criterion has been used by Alexander and Leigh (1997), etc.

covariance matrix of the interest rates using GARCH models. These results not only validate the methodology proposed in this paper, but they also point out that the use of EWMA models for calculating VaR yields superior results to those obtained using GARCH models.

References

Alexander, C.O. and C.T. Leigh (1997) "On the covariance Matrices used in Value at Risk Models", Journal of Derivatives, 4,3, 50-62.

Alexander, C.O. (2001), "Orthogonal GARCH," Mastering Risk (C.O. Alexander, Ed.) Volume 2. Financial Times – Prentice Hall, 21-38.

Basel Committee on Banking Supervision (1996), Amendment to the Capital Accord to Incorporate Market Risk.

Benito, S,. and Novales, A., (2005): "A factor analysis of volatility across the term structure: the spanish case". Documento de trabajo nº0502 del Departamento de Fundamentos de Análisis Económico II de la UCM.

Gento, P. (2000), "Un enfoque alternativo para el cálculo del valor en riesgo en carteras de renta fija", Documentos de Trabajo del Área de Teoría Económica y Economía Financiera de la Universidad de Castilla-La Mancha, 1/2000/3.

Gento, P. (2001), "Comparación entre modelos alternativos para la estimación del riesgo", Documentos de Trabajo del Área de Teoría Económica y Economía Financiera de la Universidad de Castilla-La Mancha, 1/2001/1.

García-Donato, G., Gento, P. and Ortega, J.F. (2001), "Normal versus student in measuring value at risk. An empirical bayesian overview", Documentos de Trabajo del Área de Teoría Económica y Economía Financiera de la Universidad de Castilla-La Mancha, 1/2001/4.

Gómez, I. (1999), "Aproximación al riesgo de precio de un activo de renta fija a través de un modelo de duración multifactorial paramétrico". VII Foro de Finanzas, Valencia.

Gómez, I. (1998), "Estructura Temporal de Tipos de Interés en el Mercado Español de Deuda del Estado: Caracterización, Dinámica e Implicaciones en Nuevas Herramientas para la Gestión de Carteras de Renta Fija", Tesis Doctoral, Huelva.

Jorion, P. (2000), *Value at Risk: The New Benchmark for Managing Financial Risk*, published by McGraw-Hill.

Morgan, J.P. (1995), "RiskMetrics Technical Document", 3d ed. New York.

Kevin, D.(2002), Measuring Market Risk, published by John Wiley & Sons Ltd.

Kupiec, P. (1995), "Techniques for verifying the accuracy of risk measurement models", *Journal of Derivatives*, 2, 73-84.

Ljung, G.M. and G.E.P. Box (1978), "On a Measure of Lack Fit in Time Series Models. Biometrica, 65(2) pp 297-303.

Nelson, C.R. and Siegel, A.F. (1987), "Parsimonious modelling of yield curves", Journal of Business, 60 (4), 473-489.

Appendix

The market value of the portfolio j at the moment t, which denote $V_{j,t}$, depend on k zero coupon interest rate to different maturities, as you can see in equation (1):

$$V_{j,t} = f(r_t(1), r_t(2), r_t(3), \ldots, r_t(k)) \qquad (1)$$

where f is a general function and $r_t(m)$ is the zero coupon interest rate at maturity m, ($m = 1,..k$) . From the equation (1) the portfolio value changes can be written as follow

$$dV_{j,t} = D_{j,t} dr_t \qquad (2)$$

where: $D_{j,t} = \left[\frac{\partial V_{j,t}}{\partial r_{1,t}}, \frac{\partial V_{j,t}}{\partial r_{2,t}}, \frac{\partial V_{j,t}}{\partial r_{3,t}}, \ldots, \frac{\partial V_{j,t}}{\partial r_{k,t}} \right]$ and $dr_t = [dr_t(1), dr_t(2), dr_t(3) \ldots . dr_t(k)]'$

Taking variances to both side of the equation (2) we get the following expression

$$\sigma^2{}_{\mathrm{dV}_{\mathrm{j,t}}} = \mathrm{D}_{\mathrm{j,t}} \Sigma_{\mathrm{dr}_{\mathrm{t}}} \mathrm{D}'_{\mathrm{j,t}} \qquad (3)$$

where $\sigma^2{}_{dV_{j,t}}$ represent the variance of the portfolio value changes j and Σ_{dr_t} is a matrix $k \times k$ which represent the variance-covariance matrix of the vector of the interest rate changes.

When the portfolio value depends on a very large set vector of interest rate, the dimension of Σ_{dr_t} will be high, so that to estimate this matrix we will find a dimension problem. This problem can be solved whether, as in others areas of fixed income portfolio, we assume that the structure term interest rate (TSIR) can be explained by a few number of variables o factors.

Taking the Nelson and Siegel model as starting point to explain the TSIR, the interest rate changes we can be estimated using the following expression

$$dr_t \approx G_t d\beta \qquad (4)$$

where:

$$dr_t = [dr_t(1), dr_t(2), dr_t(3) \ldots . dr_t(k)]', \qquad d\beta_t = (d\beta_{0,t}, d\beta_{1,t}, d\beta_{2,t} \ldots . d\tau_t)'$$

$$G_t = \begin{pmatrix} \frac{\partial r_t(1)}{\partial \beta_0} & \frac{\partial r_t(1)}{\partial \beta_1} & \frac{\partial r_t(1)}{\partial \beta_2} & \frac{\partial r_t(1)}{\partial \tau} \\ \frac{\partial r_t(2)}{\partial \beta_0} & \frac{\partial r_t(2)}{\partial \beta_1} & \frac{\partial r_t(2)}{\partial \beta_2} & \frac{\partial r_t(2)}{\partial \tau} \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial r_t(k)}{\partial \beta_0} & \frac{\partial r_t(k)}{\partial \beta_1} & \frac{\partial r_t(k)}{\partial \beta_2} & \frac{\partial r_t(k)}{\partial \tau} \end{pmatrix}$$

In the context of this model, and using expression (4), we can calculate the variance-covariance matrix of a vector of changes in the k interest rates using the following expression:

$$\Sigma_{\boldsymbol{dr_t}} = \boldsymbol{G_t} \Psi \boldsymbol{G_t'} \tag{5}$$

where:

$$\Psi_t = \begin{bmatrix} \mathrm{var}(\beta_{0,t}) & \mathrm{cov}(\beta_{0,t}\beta_{1,t}) & \mathrm{cov}(\beta_{0,t}\beta_{2,t}) & \mathrm{cov}(\beta_{0,t}\tau_t) \\ & \mathrm{var}(\beta_{1,t}) & \mathrm{cov}(\beta_{1,t}\beta_{2,t}) & \mathrm{cov}(\beta_{1,t}\tau_t) \\ & & \mathrm{var}(\beta_{2,t}) & \mathrm{cov}(\beta_{2,t}\tau_t) \\ & & & \mathrm{var}(\tau_t) \end{bmatrix}$$

At this point we note that we have arrived at an important simplification in the dimension of the variance-covariance matrix that we need to estimate. Note that for a vector of k interest rates, instead of having to estimate $k(k+1)/2$ variances and covariances, we only need to estimate 10 second order moments. However, the problem associated with the difficulty of the estimation of the covariances still remains.

But we can still simplify the calculation of the variance-covariance matrix even further, by applying principal components to the vector of the changes in the parameters ($d\beta_t$). In this way, the vector of changes in the parameters of the model of Nelson and Siegel (1987) can be expressed as:

$$d\beta_t = A F_t \tag{6}$$

$$\mathrm{F_t} = \left[\mathrm{f}_{1,t}, \mathrm{f}_{2,t}, \mathrm{f}_{3,t}, \mathrm{f}_{4,t}\right]' \qquad \text{and} \qquad \mathrm{A} = \begin{bmatrix} a^1_{\beta_0} & a^2_{\beta_0} & a^3_{\beta_0} & a^{41}_{\beta_0} \\ a^1_{\beta_1} & a^2_{\beta_1} & a^3_{\beta_1} & a^4_{\beta_1} \\ a^1_{\beta_2} & a^2_{\beta_2} & a^3_{\beta_2} & a^4_{\beta_2} \\ a^1_{\tau} & a^2_{\tau} & a^3_{\tau} & a^4_{\tau} \end{bmatrix}$$

where F_t is the vector of principal components associated with the vector $d\beta_t$ and A is the matrix of constants that form the eigenvectors associated with each one of the four eigenvalues of the variance-covariance matrix of the changes in the parameters of the Nelson and Siegel model ($d\beta_t$).

Substituting equation (6) into (4) we get equation (7):

$$\mathrm{dr}_t \approx \mathrm{G}^*{}_t \mathrm{F}_t \qquad (7)$$

where $\mathrm{G}_t^* = \mathrm{G}_t \mathrm{A}$. G_t is a matrix $\mathrm{k} \times 4$ and A is a matrix 4×4. Both matrixes were defined before.

Taking variances at both side of the equation (7) we get the following expression

$$\Sigma_{\mathrm{dr}_t} = \mathrm{G}_t^* \Omega \mathrm{G}_t^{*'} \qquad (8)$$

where Ω_t is the variance-covariance matrix of the principal component of the parameters changes, which is a diagonal matrix because the principal component are uncorrelated

$$\Omega_t^{'} = \begin{pmatrix} \sigma_{f_{1,t}}^2 & 0 & 0 & 0 \\ 0 & \sigma_{f_{2,t}}^2 & 0 & 0 \\ 0 & 0 & \sigma_{f_{3,t}}^2 & 0 \\ 0 & 0 & 0 & \sigma_{f_{4,t}}^2 \end{pmatrix}$$

Illustration 1. Comparing the changes of interest rates observed with the estimated changes (equation (5)).

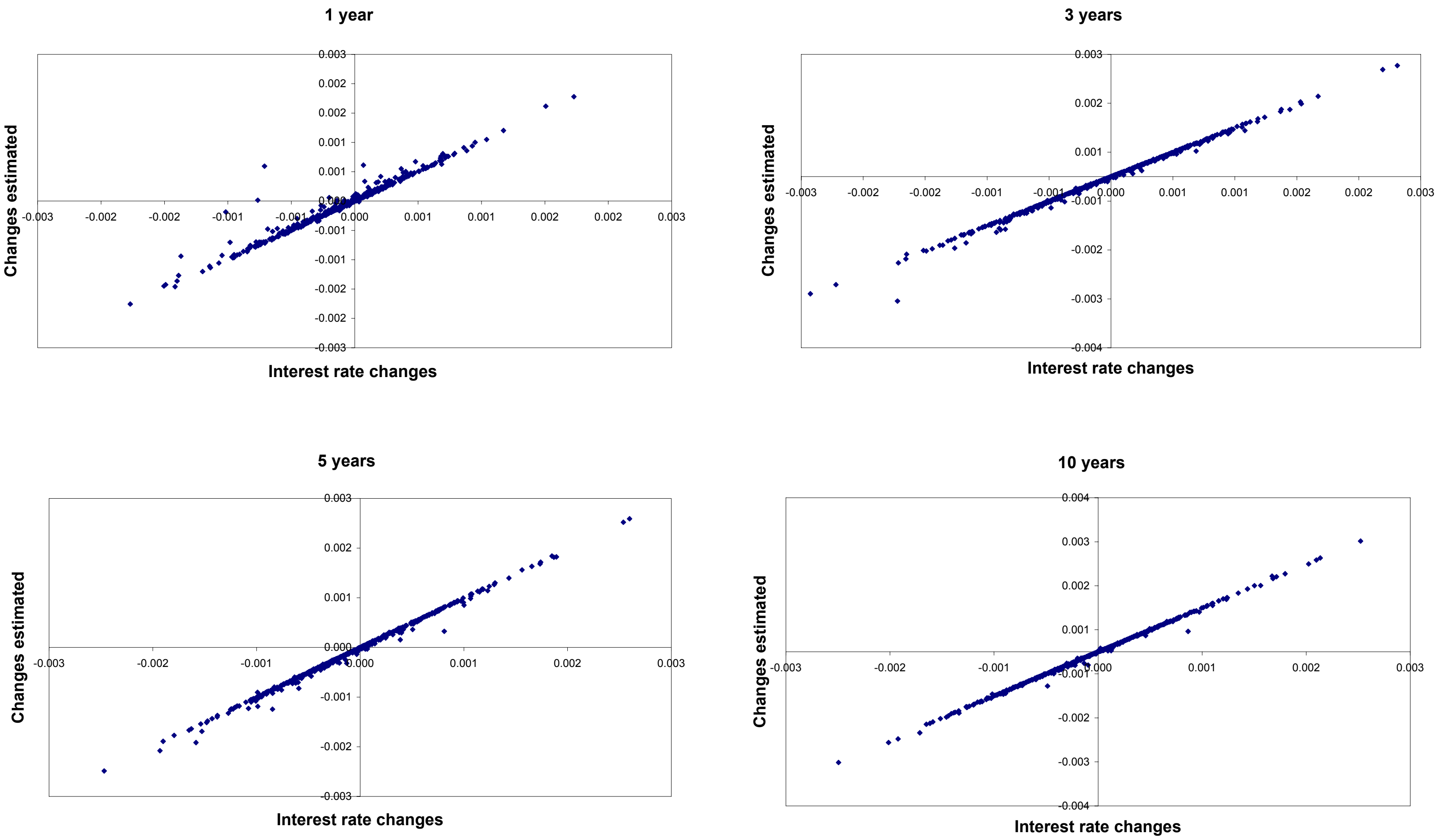


Illustration 2. Comparing the variance of changes of interest rate: Direct and indirect estimation using exponentially weighted moving average model.

Figure 1(a). Conditional Standar Deviation, 1 year

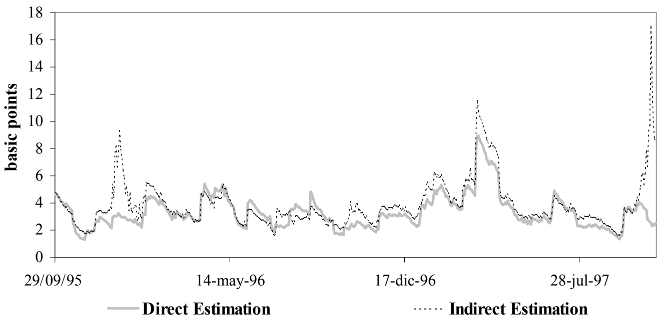


Figure 1(b). 1 year

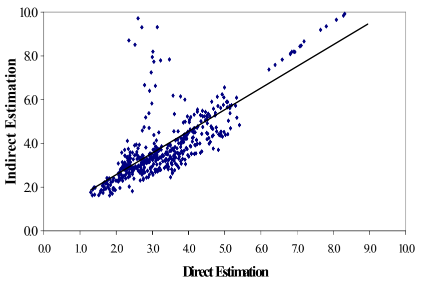


Figure 2(a). Conditional Standar Deviation, 3 years

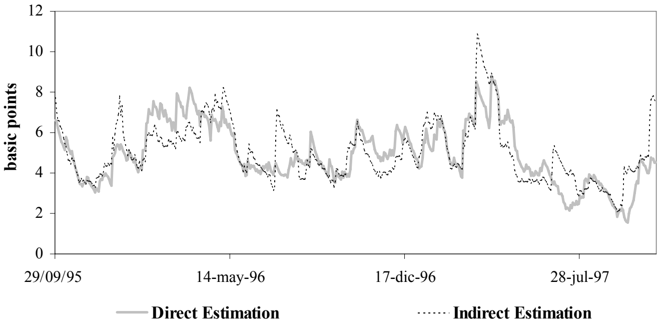


Figure 2(b). 3 years

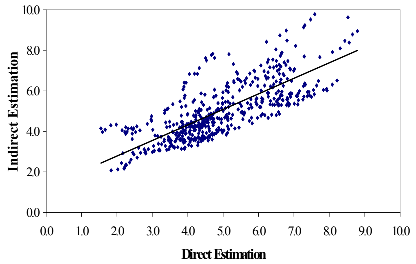


Figure 3(a). Conditional Standar Deviation, 5 years

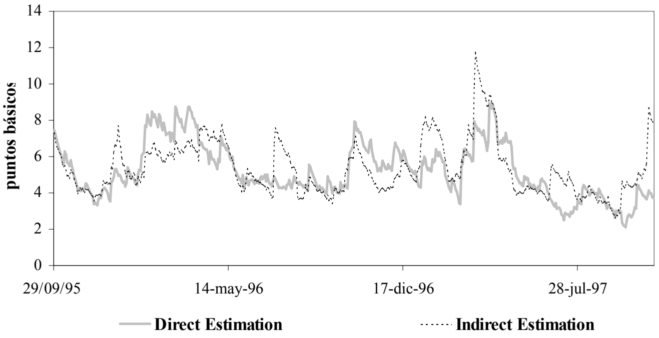


Figure 4(b). 10 years

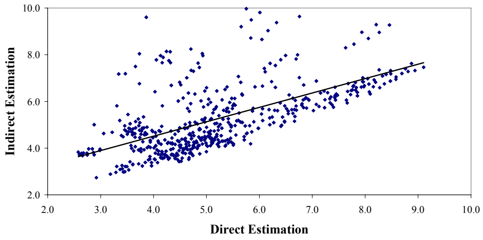


Figure 4(a). Conditional Standar Deviation, 10 years

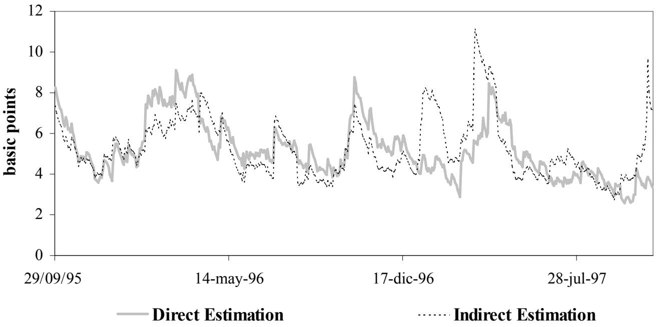


Figure 4(b). 10 years

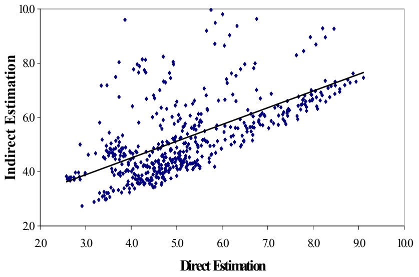


Illustration 3. Comparing the variance of changes of interest rate: Direct and indirect estimation using GARCH model.

Figure 1(a). Conditional Standar Deviation, 1 year

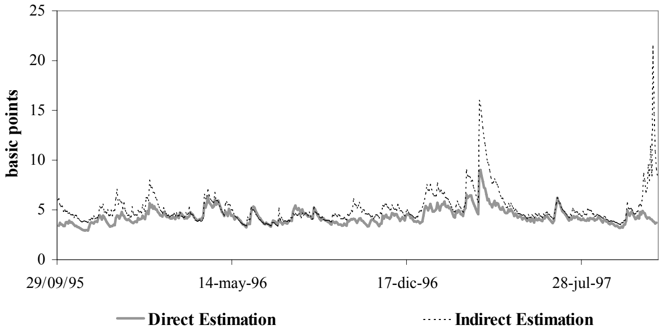


Figure 1(b). 1 year

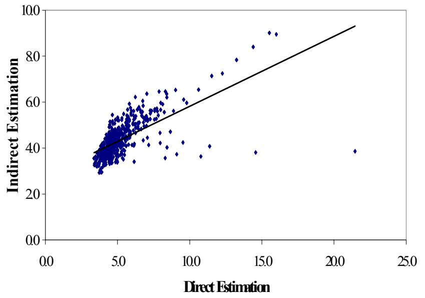


Figure 2(a). Conditional Standar Deviation, 3 years

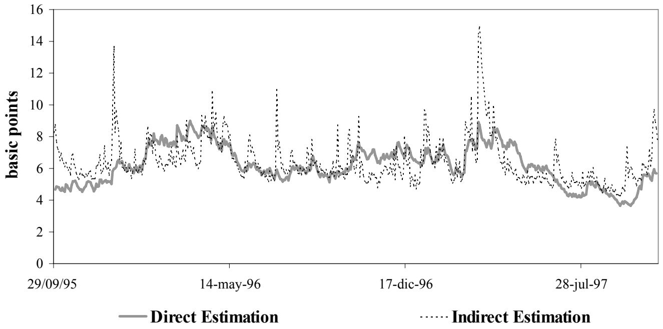


Figure 2(b). 1 year

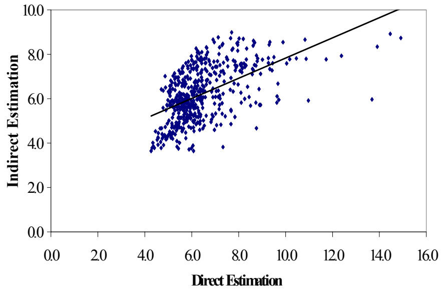


Figure 3(a). Conditional Standar Deviation, 5 years

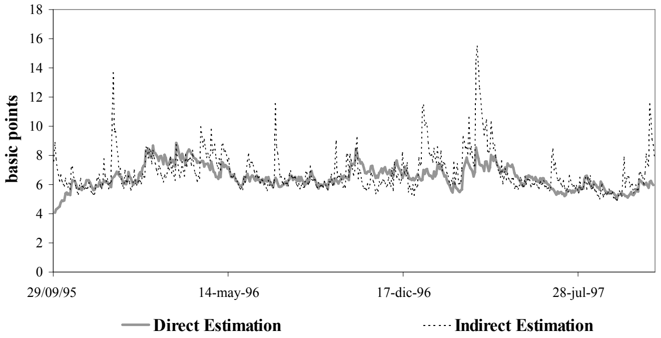


Figure 3(b). 5 years

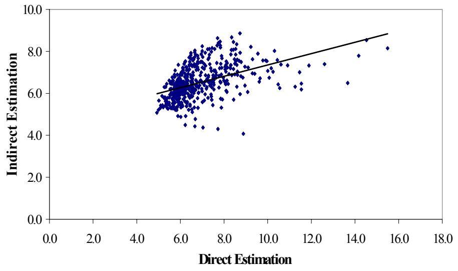


Figure 4(a). Conditional Standar Deviation, 10 years

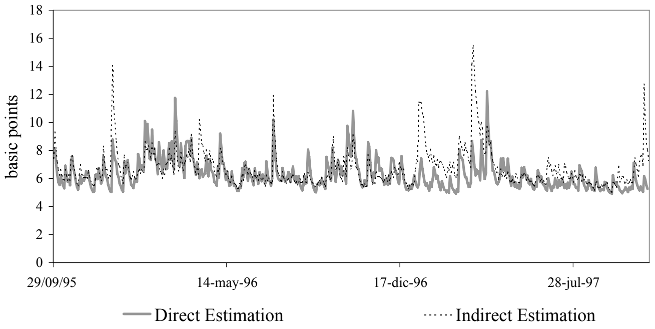


Figure 4(b). 10 years

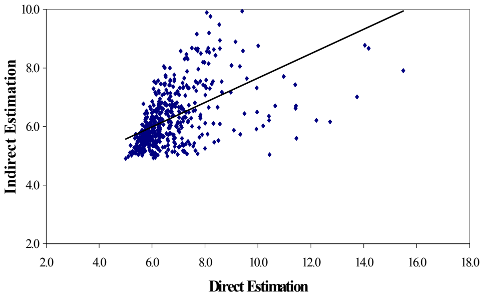
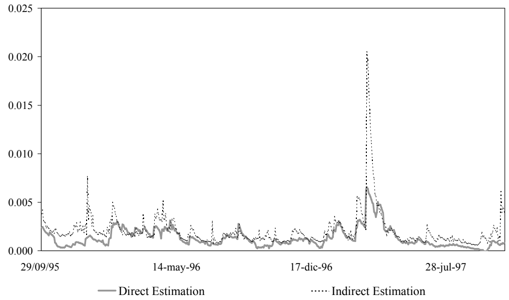


Illustration 4. Comparing the covariance between interest rate: Direct and indirect estimation using exponentially weighted moving average model.

Figura 1(a). Conditional Covariance, 1 year - 3 years



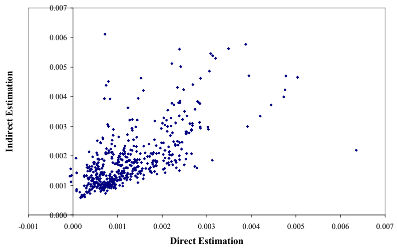
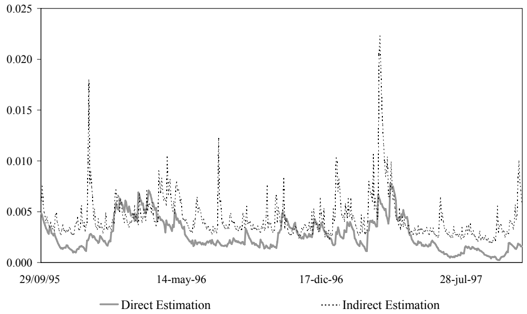


Figure 1(b). 1 year - 3 years

Figura 2(a). Conditional Covariance, 3 years - 5 years



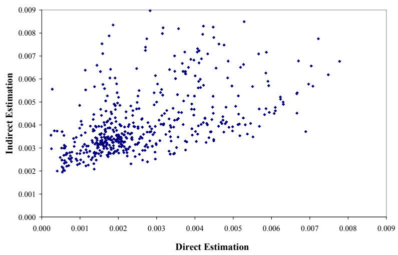
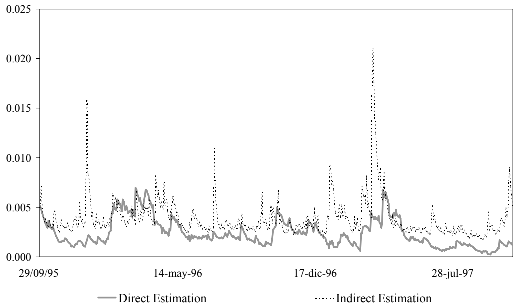


Figure 1(b). 3 year - 5 years

Figura 3(a). Conditional Covariance, 3 years - 10 years



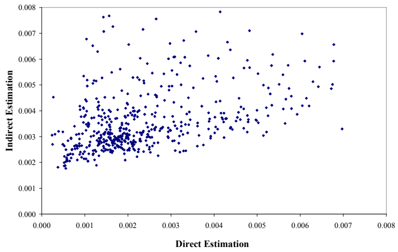
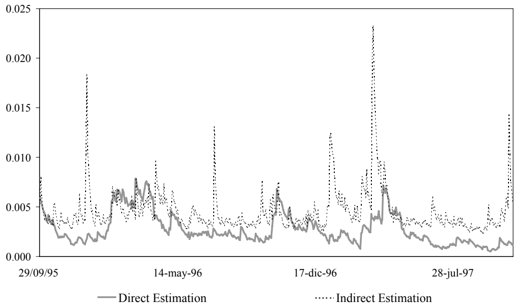


Figure 1(b). 3 year - 10 years

Figura 4(a). Conditional Covariance, 5 years - 10 years



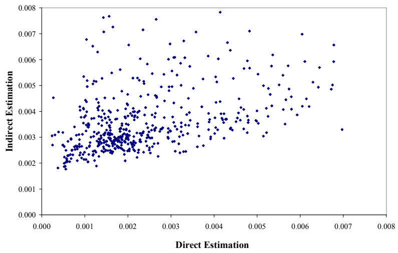
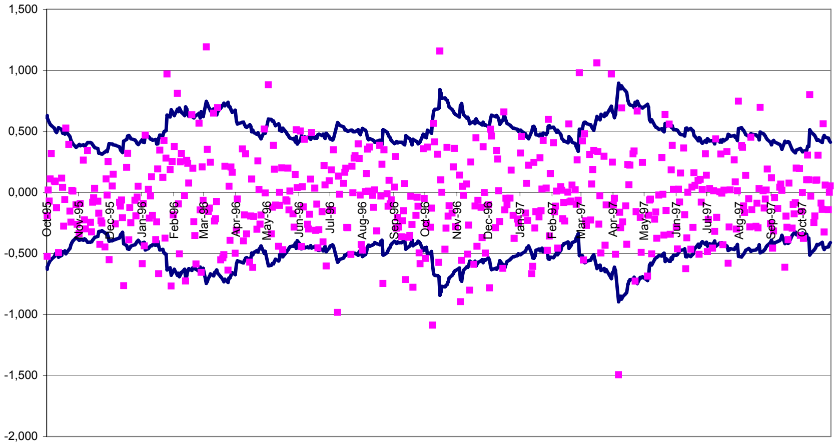
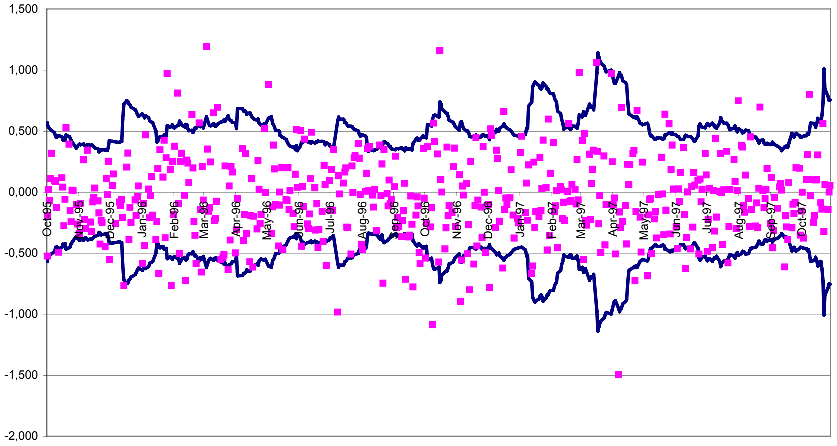
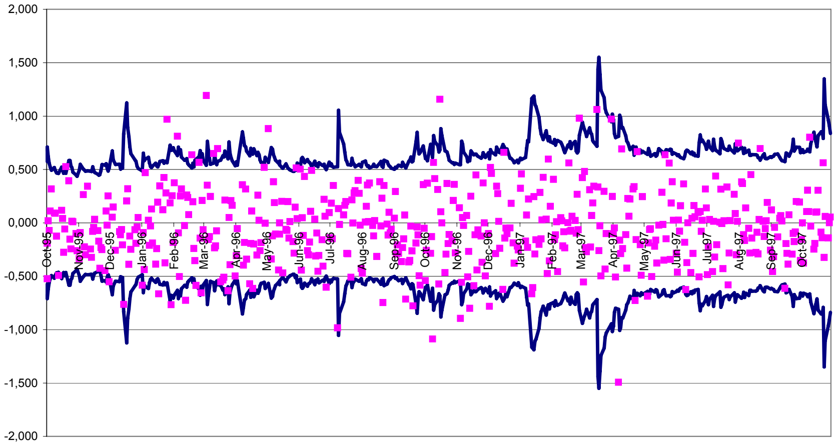


Figure 1(b). 3 year - 10 years

Illustration 5. The 5% one day VaR for a 10 year portfolio. Direct estimation using an exponentially weighted moving average model, VaR_D_EWMA(5%), indirect estimation using an exponentially weighted moving average model, VaR_I_EWMA(5%), and indirect estimation using a GARCH model, VaR_I_GARCH(5%).


5% one day VaR. Portfolio at 10 years.
VaR_D_EWMA (Riskmetrics)


5% one day VaR. Portfolio at 10 years.
VaR_I_EWMA


5% one day VaR. Portfolio at 10 years.
VaR_I_GARCH

FUNDACIÓN DE LAS CAJAS DE AHORROS

DOCUMENTOS DE TRABAJO

Últimos números publicados

159/2000 Participación privada en la construcción y explotación de carreteras de peaje
Ginés de Rus, Manuel Romero y Lourdes Trujillo

160/2000 Errores y posibles soluciones en la aplicación del *Value at Risk*
Mariano González Sánchez

161/2000 Tax neutrality on saving assets. The spahish case before and after the tax reform
Cristina Ruza y de Paz-Curbera

162/2000 Private rates of return to human capital in Spain: new evidence
F. Barceinas, J. Oliver-Alonso, J.L. Raymond y J.L. Roig-Sabaté

163/2000 El control interno del riesgo. Una propuesta de sistema de límites
riesgo neutral
Mariano González Sánchez

164/2001 La evolución de las políticas de gasto de las Administraciones Públicas en los años 90
Alfonso Utrilla de la Hoz y Carmen Pérez Esparrells

165/2001 Bank cost efficiency and output specification
Emili Tortosa-Ausina

166/2001 Recent trends in Spanish income distribution: A robust picture of falling income inequality
Josep Oliver-Alonso, Xavier Ramos y José Luis Raymond-Bara

167/2001 Efectos redistributivos y sobre el bienestar social del tratamiento de las cargas familiares en el nuevo IRPF
Nuria Badenes Plá, Julio López Laborda, Jorge Onrubia Fernández

168/2001 The Effects of Bank Debt on Financial Structure of Small and Medium Firms in some European Countries
Mónica Melle-Hernández

169/2001 La política de cohesión de la UE ampliada: la perspectiva de España
Ismael Sanz Labrador

170/2002 Riesgo de liquidez de Mercado
Mariano González Sánchez

171/2002 Los costes de administración para el afiliado en los sistemas de pensiones basados en cuentas de capitalización individual: medida y comparación internacional.
José Enrique Devesa Carpio, Rosa Rodríguez Barrera, Carlos Vidal Meliá

172/2002 La encuesta continua de presupuestos familiares (1985-1996): descripción, representatividad y propuestas de metodología para la explotación de la información de los ingresos y el gasto.
Llorenc Pou, Joaquín Alegre

173/2002 Modelos paramétricos y no paramétricos en problemas de concesión de tarjetas de credito.
Rosa Puertas, María Bonilla, Ignacio Olmeda

174/2002 Mercado único, comercio intra-industrial y costes de ajuste en las manufacturas españolas.
José Vicente Blanes Cristóbal

175/2003 La Administración tributaria en España. Un análisis de la gestión a través de los ingresos y de los gastos.
Juan de Dios Jiménez Aguilera, Pedro Enrique Barrilao González

176/2003 The Falling Share of Cash Payments in Spain.
Santiago Carbó Valverde, Rafael López del Paso, David B. Humphrey
Publicado en "Moneda y Crédito" nº 217, pags. 167-189.

177/2003 Effects of ATMs and Electronic Payments on Banking Costs: The Spanish Case.
Santiago Carbó Valverde, Rafael López del Paso, David B. Humphrey

178/2003 Factors explaining the interest margin in the banking sectors of the European Union.
Joaquín Maudos y Juan Fernández Guevara

179/2003 Los planes de stock options para directivos y consejeros y su valoración por el mercado de valores en España.
Mónica Melle Hernández

180/2003 Ownership and Performance in Europe and US Banking – A comparison of Commercial, Co-operative & Savings Banks.
Yener Altunbas, Santiago Carbó y Phil Molyneux

181/2003 The Euro effect on the integration of the European stock markets.
Mónica Melle Hernández

182/2004 In search of complementarity in the innovation strategy: international R&D and external knowledge acquisition.
Bruno Cassiman, Reinhilde Veugelers

183/2004 Fijación de precios en el sector público: una aplicación para el servicio municipal de suministro de agua.
Mª Ángeles García Valiñas

184/2004 Estimación de la economía sumergida es España: un modelo estructural de variables latentes.
Ángel Alañón Pardo, Miguel Gómez de Antonio

185/2004 Causas políticas y consecuencias sociales de la corrupción.
Joan Oriol Prats Cabrera

186/2004 Loan bankers' decisions and sensitivity to the audit report using the belief revision model.
Andrés Guiral Contreras and José A. Gonzalo Angulo

187/2004 El modelo de Black, Derman y Toy en la práctica. Aplicación al mercado español.
Marta Tolentino García-Abadillo y Antonio Díaz Pérez

188/2004 Does market competition make banks perform well?.
Mónica Melle

189/2004 Efficiency differences among banks: external, technical, internal, and managerial
Santiago Carbó Valverde, David B. Humphrey y Rafael López del Paso

190/2004	Una aproximación al análisis de los costes de la esquizofrenia en españa: los modelos jerárquicos bayesianos F. J. Vázquez-Polo, M. A. Negrín, J. M. Cavasés, E. Sánchez y grupo RIRAG
191/2004	Environmental proactivity and business performance: an empirical analysis Javier González-Benito y Óscar González-Benito
192/2004	Economic risk to beneficiaries in notional defined contribution accounts (NDCs) Carlos Vidal-Meliá, Inmaculada Domínguez-Fabian y José Enrique Devesa-Carpio
193/2004	Sources of efficiency gains in port reform: non parametric malmquist decomposition tfp index for Mexico Antonio Estache, Beatriz Tovar de la Fé y Lourdes Trujillo
194/2004	Persistencia de resultados en los fondos de inversión españoles Alfredo Ciriaco Fernández y Rafael Santamaría Aquilué
195/2005	El modelo de revisión de creencias como aproximación psicológica a la formación del juicio del auditor sobre la gestión continuada Andrés Guiral Contreras y Francisco Esteso Sánchez
196/2005	La nueva financiación sanitaria en España: descentralización y prospectiva David Cantarero Prieto
197/2005	A cointegration analysis of the Long-Run supply response of Spanish agriculture to the common agricultural policy José A. Mendez, Ricardo Mora y Carlos San Juan
198/2005	¿Refleja la estructura temporal de los tipos de interés del mercado español preferencia por la liquidez? Magdalena Massot Perelló y Juan M. Nave
199/2005	Análisis de impacto de los Fondos Estructurales Europeos recibidos por una economía regional: Un enfoque a través de Matrices de Contabilidad Social M. Carmen Lima y M. Alejandro Cardenete
200/2005	Does the development of non-cash payments affect monetary policy transmission? Santiago Carbó Valverde y Rafael López del Paso
201/2005	Firm and time varying technical and allocative efficiency: an application for port cargo handling firms Ana Rodríguez-Álvarez, Beatriz Tovar de la Fe y Lourdes Trujillo
202/2005	Contractual complexity in strategic alliances Jeffrey J. Reuer y Africa Ariño
203/2005	Factores determinantes de la evolución del empleo en las empresas adquiridas por opa Nuria Alcalde Fradejas y Inés Pérez-Soba Aguilar
204/2005	Nonlinear Forecasting in Economics: a comparison between Comprehension Approach versus Learning Approach. An Application to Spanish Time Series Elena Olmedo, Juan M. Valderas, Ricardo Gimeno and Lorenzo Escot

205/2005 Precio de la tierra con presión urbana: un modelo para España
Esther Decimavilla, Carlos San Juan y Stefan Sperlich

206/2005 Interregional migration in Spain: a semiparametric analysis
Adolfo Maza y José Villaverde

207/2005 Productivity growth in European banking
Carmen Murillo-Melchor, José Manuel Pastor y Emili Tortosa-Ausina

208/2005 Explaining Bank Cost Efficiency in Europe: Environmental and Productivity Influences.
Santiago Carbó Valverde, David B. Humphrey y Rafael López del Paso

209/2005 La elasticidad de sustitución intertemporal con preferencias no separables intratemporalmente: los casos de Alemania, España y Francia.
Elena Márquez de la Cruz, Ana R. Martínez Cañete y Inés Pérez-Soba Aguilar

210/2005 Contribución de los efectos tamaño, book-to-market y momentum a la valoración de activos: el caso español.
Begoña Font-Belaire y Alfredo Juan Grau-Grau

211/2005 Permanent income, convergence and inequality among countries
José M. Pastor and Lorenzo Serrano

212/2005 The Latin Model of Welfare: Do 'Insertion Contracts' Reduce Long-Term Dependence?
Luis Ayala and Magdalena Rodríguez

213/2005 The effect of geographic expansion on the productivity of Spanish savings banks
Manuel Illueca, José M. Pastor and Emili Tortosa-Ausina

214/2005 Dynamic network interconnection under consumer switching costs
Ángel Luis López Rodríguez

215/2005 La influencia del entorno socioeconómico en la realización de estudios universitarios: una aproximación al caso español en la década de los noventa
Marta Rahona López

216/2005 The valuation of spanish ipos: efficiency analysis
Susana Álvarez Otero

217/2005 On the generation of a regular multi-input multi-output technology using parametric output distance functions
Sergio Perelman and Daniel Santin

218/2005 La gobernanza de los procesos parlamentarios: la organización industrial del congreso de los diputados en España
Gonzalo Caballero Miguez

219/2005 Determinants of bank market structure: Efficiency and political economy variables
Francisco González

220/2005 Agresividad de las órdenes introducidas en el mercado español: estrategias, determinantes y medidas de performance
David Abad Díaz

221/2005	Tendencia post-anuncio de resultados contables: evidencia para el mercado español Carlos Forner Rodríguez, Joaquín Marhuenda Fructuoso y Sonia Sanabria García
222/2005	Human capital accumulation and geography: empirical evidence in the European Union Jesús López-Rodríguez, J. Andrés Faíña y Jose Lopez Rodríguez
223/2005	Auditors' Forecasting in Going Concern Decisions: Framing, Confidence and Information Processing Waymond Rodgers and Andrés Guiral
224/2005	The effect of Structural Fund spending on the Galician region: an assessment of the 1994-1999 and 2000-2006 Galician CSFs José Ramón Cancelo de la Torre, J. Andrés Faíña and Jesús López-Rodríguez
225/2005	The effects of ownership structure and board composition on the audit committee activity: Spanish evidence Carlos Fernández Méndez and Rubén Arrondo García
226/2005	Cross-country determinants of bank income smoothing by managing loan loss provisions Ana Rosa Fonseca and Francisco González
227/2005	Incumplimiento fiscal en el irpf (1993-2000): un análisis de sus factores determinantes Alejandro Estellér Moré
228/2005	Region versus Industry effects: volatility transmission Pilar Soriano Felipe and Francisco J. Climent Diranzo
229/2005	Concurrent Engineering: The Moderating Effect Of Uncertainty On New Product Development Success Daniel Vázquez-Bustelo and Sandra Valle
230/2005	On zero lower bound traps: a framework for the analysis of monetary policy in the ‘age’ of central banks Alfonso Palacio-Vera
231/2005	Reconciling Sustainability and Discounting in Cost Benefit Analysis: a methodological proposal M. Carmen Almansa Sáez and Javier Calatrava Requena
232/2005	Can The Excess Of Liquidity Affect The Effectiveness Of The European Monetary Policy? Santiago Carbó Valverde and Rafael López del Paso
233/2005	Inheritance Taxes In The Eu Fiscal Systems: The Present Situation And Future Perspectives. Miguel Angel Barberán Lahuerta
234/2006	Bank Ownership And Informativeness Of Earnings. Víctor M. González
235/2006	Developing A Predictive Method: A Comparative Study Of The Partial Least Squares Vs Maximum Likelihood Techniques. Waymond Rodgers, Paul Pavlou and Andres Guiral.
236/2006	Using Compromise Programming for Macroeconomic Policy Making in a General Equilibrium Framework: Theory and Application to the Spanish Economy. Francisco J. André, M. Alejandro Cardenete y Carlos Romero.

237/2006	Bank Market Power And Sme Financing Constraints. Santiago Carbó-Valverde, Francisco Rodríguez-Fernández y Gregory F. Udell.
238/2006	Trade Effects Of Monetary Agreements: Evidence For Oecd Countries. Salvador Gil-Pareja, Rafael Llorca-Vivero y José Antonio Martínez-Serrano.
239/2006	The Quality Of Institutions: A Genetic Programming Approach. Marcos Álvarez-Díaz y Gonzalo Caballero Miguez.
240/2006	La interacción entre el éxito competitivo y las condiciones del mercado doméstico como determinantes de la decisión de exportación en las Pymes. Francisco García Pérez.
241/2006	Una estimación de la depreciación del capital humano por sectores, por ocupación y en el tiempo. Inés P. Murillo.
242/2006	Consumption And Leisure Externalities, Economic Growth And Equilibrium Efficiency. Manuel A. Gómez.
243/2006	Measuring efficiency in education: an analysis of different approaches for incorporating non-discretionary inputs. Jose Manuel Cordero-Ferrera, Francisco Pedraja-Chaparro y Javier Salinas-Jiménez
244/2006	Did The European Exchange-Rate Mechanism Contribute To The Integration Of Peripheral Countries?. Salvador Gil-Pareja, Rafael Llorca-Vivero y José Antonio Martínez-Serrano
245/2006	Intergenerational Health Mobility: An Empirical Approach Based On The Echp. Marta Pascual and David Cantarero
246/2006	Measurement and analysis of the Spanish Stock Exchange using the Lyapunov exponent with digital technology. Salvador Rojí Ferrari and Ana Gonzalez Marcos
247/2006	Testing For Structural Breaks In Variance Withadditive Outliers And Measurement Errors. Paulo M.M. Rodrigues and Antonio Rubia
248/2006	The Cost Of Market Power In Banking: Social Welfare Loss Vs. Cost Inefficiency. Joaquín Maudos and Juan Fernández de Guevara
249/2006	Elasticidades de largo plazo de la demanda de vivienda: evidencia para España (1885-2000). Desiderio Romero Jordán, José Félix Sanz Sanz y César Pérez López
250/2006	Regional Income Disparities in Europe: What role for location?. Jesús López-Rodríguez and J. Andrés Faíña
251/2006	Funciones abreviadas de bienestar social: Una forma sencilla de simultanear la medición de la eficiencia y la equidad de las políticas de gasto público. Nuria Badenes Plá y Daniel Santín González
252/2006	“The momentum effect in the Spanish stock market: Omitted risk factors or investor behaviour?”. Luis Muga and Rafael Santamaría
253/2006	Dinámica de precios en el mercado español de gasolina: un equilibrio de colusión tácita. Jordi Perdiguero García

254/2006 Desigualdad regional en España: renta permanente versus renta corriente.
José M.Pastor, Empar Pons y Lorenzo Serrano

255/2006 Environmental implications of organic food preferences: an application of the impure public goods model.
Ana Maria Aldanondo-Ochoa y Carmen Almansa-Sáez

256/2006 Family tax credits versus family allowances when labour supply matters: Evidence for Spain.
José Felix Sanz-Sanz, Desiderio Romero-Jordán y Santiago Álvarez-García

257/2006 La internacionalización de la empresa manufacturera española: efectos del capital humano genérico y específico.
José López Rodríguez

258/2006 Evaluación de las migraciones interregionales en España, 1996-2004.
María Martínez Torres

259/2006 Efficiency and market power in Spanish banking.
Rolf Färe, Shawna Grosskopf y Emili Tortosa-Ausina.

260/2006 Asimetrías en volatilidad, beta y contagios entre las empresas grandes y pequeñas cotizadas en la bolsa española.
Helena Chuliá y Hipòlit Torró.

261/2006 Birth Replacement Ratios: New Measures of Period Population Replacement.
José Antonio Ortega.

262/2006 Accidentes de tráfico, víctimas mortales y consumo de alcohol.
José Mª Arranz y Ana I. Gil.

263/2006 Análisis de la Presencia de la Mujer en los Consejos de Administración de las Mil Mayores Empresas Españolas.
Ruth Mateos de Cabo, Lorenzo Escot Mangas y Ricardo Gimeno Nogués.

264/2006 Crisis y Reforma del Pacto de Estabilidad y Crecimiento. Las Limitaciones de la Política Económica en Europa.
Ignacio Álvarez Peralta.

265/2006 Have Child Tax Allowances Affected Family Size? A Microdata Study For Spain (1996-2000).
Jaime Vallés-Giménez y Anabel Zárate-Marco.

266/2006 Health Human Capital And The Shift From Foraging To Farming.
Paolo Rungo.

267/2006 Financiación Autonómica y Política de la Competencia: El Mercado de Gasolina en Canarias.
Juan Luis Jiménez y Jordi Perdiguero.

268/2006 El cumplimiento del Protocolo de Kyoto para los hogares españoles: el papel de la imposición sobre la energía.
Desiderio Romero-Jordán y José Félix Sanz-Sanz.

269/2006 Banking competition, financial dependence and economic growth
Joaquín Maudos y Juan Fernández de Guevara

270/2006 Efficiency, subsidies and environmental adaptation of animal farming under CAP
Werner Kleinhanß, Carmen Murillo, Carlos San Juan y Stefan Sperlich

271/2006	Interest Groups, Incentives to Cooperation and Decision-Making Process in the European Union A. Garcia-Lorenzo y Jesús López-Rodríguez
272/2006	Riesgo asimétrico y estrategias de momentum en el mercado de valores español Luis Muga y Rafael Santamaría
273/2006	Valoración de capital-riesgo en proyectos de base tecnológica e innovadora a través de la teoría de opciones reales Gracia Rubio Martín
274/2006	Capital stock and unemployment: searching for the missing link Ana Rosa Martínez-Cañete, Elena Márquez de la Cruz, Alfonso Palacio-Vera and Inés Pérez-Soba Aguilar
275/2006	Study of the influence of the voters' political culture on vote decision through the simulation of a political competition problem in Spain Sagrario Lantarón, Isabel Lillo, Mª Dolores López and Javier Rodrigo
276/2006	Investment and growth in Europe during the Golden Age Antonio Cubel and Mª Teresa Sanchis
277/2006	Efectos de vincular la pensión pública a la inversión en cantidad y calidad de hijos en un modelo de equilibrio general Robert Meneu Gaya
278/2006	El consumo y la valoración de activos Elena Márquez y Belén Nieto
279/2006	Economic growth and currency crisis: A real exchange rate entropic approach David Matesanz Gómez y Guillermo J. Ortega
280/2006	Three measures of returns to education: An illustration for the case of Spain María Arrazola y José de Hevia
281/2006	Composition of Firms versus Composition of Jobs Antoni Cunyat
282/2006	La vocación internacional de un holding tranviario belga: la Compagnie Mutuelle de Tramways, 1895-1918 Alberte Martínez López
283/2006	Una visión panorámica de las entidades de crédito en España en la última década. Constantino García Ramos
284/2006	Foreign Capital and Business Strategies: a comparative analysis of urban transport in Madrid and Barcelona, 1871-1925 Alberte Martínez López
285/2006	Los intereses belgas en la red ferroviaria catalana, 1890-1936 Alberte Martínez López
286/2006	The Governance of Quality: The Case of the Agrifood Brand Names Marta Fernández Barcala, Manuel González-Díaz y Emmanuel Raynaud
287/2006	Modelling the role of health status in the transition out of malthusian equilibrium Paolo Rungo, Luis Currais and Berta Rivera
288/2006	Industrial Effects of Climate Change Policies through the EU Emissions Trading Scheme Xavier Labandeira and Miguel Rodríguez

289/2006	Globalisation and the Composition of Government Spending: An analysis for OECD countries Norman Gemmell, Richard Kneller and Ismael Sanz
290/2006	La producción de energía eléctrica en España: Análisis económico de la actividad tras la liberalización del Sector Eléctrico Fernando Hernández Martínez
291/2006	Further considerations on the link between adjustment costs and the productivity of R&D investment: evidence for Spain Desiderio Romero-Jordán, José Félix Sanz-Sanz and Inmaculada Álvarez-Ayuso
292/2006	Una teoría sobre la contribución de la función de compras al rendimiento empresarial Javier González Benito
293/2006	Agility drivers, enablers and outcomes: empirical test of an integrated agile manufacturing model Daniel Vázquez-Bustelo, Lucía Avella and Esteban Fernández
294/2006	Testing the parametric vs the semiparametric generalized mixed effects models María José Lombardía and Stefan Sperlich
295/2006	Nonlinear dynamics in energy futures Mariano Matilla-García
296/2006	Estimating Spatial Models By Generalized Maximum Entropy Or How To Get Rid Of W Esteban Fernández Vázquez, Matías Mayor Fernández and Jorge Rodriguez-Valez
297/2006	Optimización fiscal en las transmisiones lucrativas: análisis metodológico Félix Domínguez Barrero
298/2006	La situación actual de la banca online en España Francisco José Climent Diranzo y Alexandre Momparler Pechuán
299/2006	Estrategia competitiva y rendimiento del negocio: el papel mediador de la estrategia y las capacidades productivas Javier González Benito y Isabel Suárez González
300/2006	A Parametric Model to Estimate Risk in a Fixed Income Portfolio Pilar Abad and Sonia Benito